



Cornell University®

Big Red Math Competition
Team Round

Team ID: _____
10/25/2025 2:05pm–2:35pm

INSTRUCTIONS — PLEASE READ THIS NOW

- **Write your Team ID. ONLY captain needs to turn in.**
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos \pi}{\frac{1}{2}}$ can be simplified to -2 .
- **You have 30 minutes to complete this exam.**
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources. You **CAN** collaborate with your teammates.

Please sign below to indicate that you have read and agree to these instructions.

Signature of Captain, Print Name: _____

Answer Sheet:

- | | | |
|----------|----------|-----------|
| 1. _____ | 5. _____ | 9. _____ |
| 2. _____ | 6. _____ | 10. _____ |
| 3. _____ | 7. _____ | |
| 4. _____ | 8. _____ | |
-

Please don't mark anything below on the page, it's for official use only.

Grader 1 Name:

Grader 2 Name:

Grader 3 Name:

Tally:

Tally:

Tally:

Score:

Score:

Score:

1. Find the sum of all numbers between 1 and 999, inclusive, that have exactly two digits equal to 5.
2. Mia chooses an integer uniformly at random between 1 and 100, inclusive. Vincent and Jules are playing a game where they each choose an integer between 1 and 100, inclusive, and the winner is the person whose integer is closest to Mia's integer. If both of their integers are equally close to Mia's, then Mia decides the winner uniformly at random. Suppose Jules knows what Vincent's integer is before he chooses his own. What is the probability that Jules wins if he plays optimally?
3. Let ABC be a triangle with incenter I and side lengths $AB = 10$, $AC = 24$ and $BC = 26$. Suppose a line through I meets segments AB and AC at P and Q , respectively. Find the minimum possible area of triangle APQ .
4. Eighty Cornell students, out of which some are freshmen and the rest are sophomore, go on a road trip. When making a line to get on the bus, a very observant professor notes that, regardless of how they stand in line, there will always be a pair of students from the same year standing with exactly 8 students between them. If there are more sophomores than freshman on the trip, what is the maximum possible number of freshmen there can be?
5. How many integers $b \geq 2$ are there such that the number $25!$ ends with at least two 0s when written in base b ?
6. Each one of the 2025 inhabitants of the old town of Ithaca is either a knight or a fraud. It is well known amongst the wise that there are at most C knights. Knights always say the truth, while frauds always lie. Every year on October 26th, Ithaca is visited by a wise who wishes to decipher the number of knights in town. The wise makes all 2025 inhabitants stand in a row, and each one of them (but the one in front) says what the person in front of them is, either a knight or a fraud. Determine the maximum value of C such that, regardless of the 2024 answers, the wise can accomplish his wish.
7. How many subsets of $\{1, 2, \dots, 12\}$ have exactly one pair of consecutive numbers?
8. What is the smallest perimeter that a triangle can have if it has a vertex on the x -axis, a vertex on the line $y = x$, and another vertex at the point $(2700, 2025)$?
9. The function $f: \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ satisfies the property that for all $m, n \in \mathbb{Z}^+$, we have

$$\gcd(f(m), n)f(mn) = f(m)f(n).$$

If the smallest possible value of $f(2025)$ is a and the largest possible value of $f(2025)$ is b , find $a + b$.

Note: $\mathbb{Z}^+$ denotes the set of positive integers.

10. Cooper is currently standing at the point 0 on the number line. After every second, he either moves one unit to the left, one unit to the right, or doesn't move at all, each occurring with probability $1/3$. What is the probability that after exactly 2025 seconds, Cooper is standing on an integer divisible by 4?